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Analytic Approximation of Integral Intensity of the Line λ 630 nm

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The deduction of the quantity relationships between the airglow and the upper atmospheric parameters provide for the complete analysis of the phenomena in this medium. The aim of this paper is to obtain the approximate analytic expressions for the integral intensity I of the red oxygen line λ 630 nm, generated in the upper atmosphere of the earth. The formulae for the volume emission rate dI/dh, obtained in [1, 2], shape the background for these expressions based on the models for dI/dh in [3]. The latter are based on the model for the neutral atmosphere CIRA-72 and the International Reference Ionosphere IRI-75. The analysis of the new models for dI/dh in agreement with CIRA-79 and IRI-79, as well as the models based on MSIS, combined with IRI (see [4]), do not change the nature of the analytic expressions. Only their digital characteristics are changed. Through the expressions thus deduced for the integral intensity I, in dependence on the parameters of the models for the neutral upper atmosphere and for the ionosphere, a possibility is provided to resolve series of direct and reverse problems — from the known parameters to determine the intensity, or from the known intensity, combined with measurements on the intensity of the line λ 135,6 nm and single measurement on the local electron density, to measure the values of the maximal electron density N_mF (or its respective critical frequency f_0F); of the height of this density h_mF and the constant A_0 of the distribution $N(h)$ in agreement with IRI — see the method in [5]. On the other hand, such expressions enable us to consider the effects of various factors determining the intensity of the line λ 630 nm, and to compare the models of the neutral atmosphere and the ionosphere.

Complete Expressions for the Intensity I_{630}

In [1] from the general expression for the volume emission rate

(1) $$\frac{dI}{dH}=\frac{KA_{630}}{A}\frac{\gamma_1 [O_2] N dh}{[1+d(h)/A][1+B(h)]}$$

approximation of the denominator is obtained

(2) $$[1+d(h)/A][1+B(h)]\approx\zeta_0(h_0F)e^{-p_1(h-h_0F)}=\zeta(h),$$

where $K=1$, $A_{630}=0{,}069\ s^{-1}$, $A=0{,}0091\ s^{-1}$, N — electron density, $d(h)$ denotes the total deactivation of collisions of excited oxygen atoms in state $O(^1D)$ with neutral nitrogen molecules and with the electrons: $B(h)\gamma_1[0_2]/\alpha_1 N+\gamma_2[N_2]/\alpha_2 N$, where γ_1 and γ_2 are respectively the coefficients of the exchange reactions between the ions of the atomic oxygen and the neutral oxygen, or the nitrogen molecules; and α_1 and α_2 are the coefficients of the dissociative recombination for ions O_2^+ and NO^+, respectively. For the values of the examined coefficients and their temperature, temporal and spatial variations see [1, 2]. In agreement with [1] at midgeographic latitudes ($\varphi=45°$) in 00^h summertime under $R=10\,(T_{ex}\approx 550°K)$ and $h_0F\approx 180$ km, we have $\zeta_0(180)\approx 347{,}4$ and $p_1\approx 0{,}4$.

In agreement with IRI-79 for $N(h)$ under $h\geq h_mF$ (further for simplification we shall use $N_mF=N_m$ and $h_mF=h_m$) we have

$$N(h)=N_m\left[A_1/\left(1+\frac{h-h_m}{A_0}\right)^{A_2}-A_3\exp\left(-A_4\frac{h-h_m}{A_0}\right)\right], \tag{3}$$

where: A_1 — constant of two fixed values (for low and high solar activity, see IRI); $A_2=2$ (at low solar activity); $A_3=3$ (at high solar activity); $A_3=A_1-1$; $A_4=A_1A_2/(A_1-A_2)-0{,}05$. It is clear that under known solar activity all the constants are known from IRI, excluding A_0, which is either taken from IRI or specified through measurements similar to those in [5]. Expressions for $N(h)$ are given in IRI for $h\leq h_mF$ which introduce series of new constants. But we use in [5] the known fact that the circummaximum area of the F-region has distribution very close to the parabolic one and is characterized with symmetry with reference to h_mF. Therefore, we use expression (3) for $h\leq h_mF$ also up to heights of about 180-240 km, and in this region in agreement with [2, 3, 4] the airglow with λ 630 nm is initiated. For the purpose we only substitute $h-h_m$ with the argument h_m-h.

We have shown in [1, 2] that $\zeta(h)$ is given with the expression (2) up to a height of about 280 km over the earth. At $h\geq 280$ km, we have $\zeta\approx 1$. Therefore, we analyse the following cases:

1. Region with $\zeta\neq 1\,(h\leq 280$ km).

This case can be divided into the following subcases, considering the location of h_mF:

1.1. $h_mF<280$ km.

The airglow occurs in three parts: a) $h_m\leq h<280$ km; b) $h\leq h_m$; c) $h\geq 280$ km. We denote with r, g and n the respective parts of the intensity

$$I=I_r+I_g+I_n. \tag{4}$$

a) $h_m\leq 280$ km

For low solar activity, the following expression for I is obtained from (1), (2), (3)

$$I_r=I_{r_1}+I_{r_2}; \tag{5}$$

$$I_{r_1}=\frac{KA_{630}}{A\zeta_0}\gamma_1 0_2 J_0(180)N_mA_1\int\limits_{h_m}^{280}\frac{e^{-(p_2-p_1)(h-180)}}{\left(1+\frac{h-h_m}{A_0}\right)^2}dh. \tag{6}$$

The exponential approximation for the altitudinal distribution of the molecular oxygen $[0_2](h)\approx[0_2]_0(180)e^{-p_2(h-180)}$ versus the possibly lowest boundary of the airglow of 180 km is considered in the yielding of (6). It is considered that, regardless to the complex nature of the temperature (and therefore altitudinal) variations of γ_1, in agreement with (1) we may consider that

$\gamma_1 \approx 1{,}60\ 10^{-11}\ \mathrm{cm}^3\ S^{-1}$ in the examined altitudinal region. It follows from (6) that

$$I_{r_1} = K_1 N_m A_0 e^{-(p_2-p_1)(h_m-A_0-180)} \left\{ \frac{-e^{A_0(p_2-p_1)U}}{U} - A_0(p_2-p_1) E_i[A_0(p_1-p_2)U] \right\}_{U=1}^{\frac{280-A_0-hm}{A_0}}, \tag{7}$$

where $K_1 = \frac{KA_{630}}{A\zeta_0} \gamma_1 [0_2]_0(180)A_1$; E_i is the generally accepted denomination of the exponential-integral function.

At high solar activity ($R \geq 100$) we have

$$I_{r_1} = K_1 N_m \int_{h_m}^{280} \frac{e^{-(p_2-p_1)(h-180)}}{\left(1+\frac{h-h_m}{A_0}\right)^3} dh; \tag{8}$$

$$I_{r_1} = K_1 N_m A_0 e^{-(p_2-p_1)(h_m-A_0-180)} \left\{ \frac{-e^{-(p_2-p_1)A_0U}}{2U^2} + \frac{(p_2-p_1)A_0 e^{-(p_2-p_1)A_0U}}{2U} + \frac{(p_2-p_1)^2 A_0^2}{2} E_i[A_0(p_1-p_2)U] \right\}_{U=1}^{\frac{280-h_m-A_0}{A}}. \tag{9}$$

The expression for I_{r_2} does not depend on the solar activity and we obtain

$$I_{r_2} = \frac{KA_{630}}{A\zeta_0} \gamma_1 [0_2]_0(180) A_3 N_m \int_{h_m}^{280} e^{-(p_2-p_1)(h-180)} e^{-A_4 \frac{h-h_m}{A_0}} dh; \tag{10}$$

$$I_{r_2} = \frac{K_2 A_0 N_m e^{-(p_2-p_1)(h_m-180)}}{(p_2-p_1)A_0+A_4} \left\{ 1 - e^{-[(p_2-p_1)A_0+A_4] \frac{280-h_m}{A_0}} \right\}, \tag{11}$$

where

$$K_2 = \frac{KA_{630}}{A\zeta_0} \gamma_1 [0_2]_0 (180) A_3. \tag{12}$$

b) $h \leq h_m$

As we have already shown, in this case we substitute in (3) the expression $h-h_m$ with h_m-h, which is yielded at low solar activity

$$I_{g_1} = K_1 N_m A_0 e^{-(p_2-p_1)(h_m+A_0-180)} \left\{ -e^{A_0(p_2-p_1)U} + A_0(p_2-p_1) E_i[A_0(p_2-p_1)U] \right\}_{U=1}^{\frac{h_m+A_0-180}{A_0}}; \tag{13}$$

$$I_{g_1} = \frac{K_2 A_0 N_m e^{-(p_2-p_1)(h_m-180)}}{A_4-(p_2-p_1)A_0} \left\{ 1 - e^{-[A_4-(p_2-p_1)] \frac{h_m-180}{A_0}} \right\}. \tag{14}$$

At high solar activity we have

$$I_{g_1} = K_1 N_m A_0 e^{-(p_2-p_1)(h_m+A_0-180)} \left\{ -\frac{e^{(p_2-p_1)A_0U}}{2U2} - \frac{A_0(p_2-p_1)e^{(p_2-p_1)A_0U}}{2U} + \frac{(p_2-p_1)^2 A_0^2}{2} E_i[A_0(p_2-p_1)U] \right\}_{U=1}^{\frac{h_m+A_0-180}{A_0}}. \tag{15}$$

I_{g_2} has the form of (14).

c) $h>280$ km

This case is treated under $\zeta=1$.

1.2. $h_mF>280$ km

In this case the total intensity is also formed in three layers: a) $180\leq h\leq 280$ km/$\zeta\neq 1$; submaximum region; b) $280\leq h\leq h_m/\zeta=1$; submaximum region, and c) $h>h_m/\zeta=1$; abovemaximum region. Cases b) and c) will be considered for $\zeta=1$. For case a) we use (13) and (14) for low and high activity respectively and substitute the bottom boundary in both cases with $(h_m+A_0-280)/A_0$.

2. $\zeta=1$ ($h>280$ km)

2.1. $h_mF<280$ km

We have again the subcases and the respective componets given by the indices r, g and n, shown in 1.1. We use for the components r and g the already deduced expressions (7) and (9), (11), (13), (14), (15). The following dependence is given for the component n

$$I_n=I_{n_1}+I_{n_2}, \tag{16}$$

where at low solar activity

$$I_{n_1}=\frac{KA_{630}}{A}\gamma_1[O_2]_0(180)N_mA_1\int\limits_{280}^{h_r}\frac{e^{-p_2(h-180)}}{\left(1+\frac{h-h_m}{A_0}\right)^2}dh; \tag{17}$$

$$I_{n_1}=K_3N_mA_0e^{-p_2(h_m-A_0-180)}\left\{\frac{-e^{-p_2A_0U}}{U}-p_2A_0E_i(-p_2A_0U)\right\}\Bigg|_{U=\frac{280+A_0-h_m}{A_0}}^{\frac{400+A_0-h_m}{A_0}}. \tag{18}$$

At high solar activity

$$I_{n_1}=K_3N_mA_0e^{-p_2(h_m-A_0-180)}\left[\frac{e^{-p_2A_0U}}{2U^2}+\frac{p_2A_0e^{-p_2A_0U}}{2U}+\frac{p_2^2A_0^2}{2}E_i(-p_2A_0U)\right]\Bigg|_{U=\frac{280+A_0-h_m}{A_0}}^{\frac{600+A_0-h_m}{A_0}}, \tag{19}$$

where $K_3=KA_{630}\gamma_1[O_2]_0(180)A_1/A$.

It is considered in (18) and (19) that in agreement with [1, 3, 4] the upper boundary of the airglow with λ 630 nm is about $h_r\approx 400$ km at low activity and about 600 km at high activity. For I_{n_2} we have

$$I_{n_2}=\frac{K_4A_0N_me^{-p_2(h_m-180)}}{p_2A_0+A_4}\left[e^{-(p_2A_0+A_4)\frac{280-h_m}{A_0}}-e^{-(p_2A_0+A_4)\frac{h_r-h_m}{A_0}}\right], \tag{20}$$

where $K_4=KA_{630}\gamma_1[O_2]_0(180)A_3/A$.

2.2. $h_mF>280$ km

In this case also three components of the indices r, g and n are available, and I_{r_1} is determined with (13) or (15) for low and high activity, respectively, but with substituted bottom boundary: instead of $U=1$, $U=\frac{h_m+A_0-280}{A_0}$ is used; expression (7) is justified for the component I_{g_1} with substituted argument — instead of $h-h_m$, h_m-h. Expressions similar to (18) of upper boundary equal to an unit are obtained. Dependences such as (18), (19) and (20) are used for the components I_n with substituted bottom boundary, namely $U=1$.

With the dependences here deduced, various ionospheric or aeronomic parameters can be determined and the respective equations are solved in graph-analytical manner similar to the method in [6] or the *regula falsi* method. The data from IRI can be used in the latter case as orientation for the search of the respective solutions.

Discussion and conclusions

According to IRI, the cases with $h_mF \leq 280$ km are quite rare. In fact, our observations on midgeographic latitudes contradict this concept since often $220 \leq h_mF \leq 280$ km, see summaries in [2, 10].

For the computing procedures it should be considered that in many cases (for example at high values of the power index in the decreasing exponential functions or at large arguments of the integral-exponential function) significant simplifications of the expressions deduced here are obtained. In agreement with the method developed in [5, 7] the expressions of the intensity of the red oxygen line, together with those of the intensity of the ultraviolet line with λ 135,6 nm and a local measurement of the electron density provide yield of parameters N_m, A_0 and h_m. This is the determination of all the parameters necessary for the computation of the maximal usable frequencies (MUF) in radiocommunications. On the other hand, the determination of the profile $N(h)$ of the electron density in this manner and its correlation with the concrete measurements of the mentioned airglow emissions and with a value of the electron density, enable the performance of improvement and updating of the IRI model. Therefore, its adequacy is enhanced.

The expressions obtained contain the unknowns N_m, h_m and A_0 of very complex relationships. But their graph-analytic solution or through the *regula falsi* method is not difficult in the application of the orientating values of the IRI, or from the ionospheric models of the CCIR.

Both through the dependences obtained here and through the formulae found for the intensity of the emission with λ 135,6 nm in [5, 8, 9] the background for the complete consideration of all the factors for the generation of the most important oxygen lines in the airglow and for the serious mathematical theory of this phenomenon in the upper atmospheres of the planets and the earth is compiled.

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Аналитическая аппроксимация интегральной интенсивности линии λ 630 nm

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(Р е з ю м е)

На основе модели нейтральной атмосферы CIRA-79 и Интернациональной референтной ионосферы IRI-79 выведены полные аналитические выражения интенсивности красной линии кислорода на длине волны λ 630 nm. Рассмотрены различные случаи и представлены полные формулы для I_{630} в зависимости от взаимного расположения высоты максимальной электронной концентрации N_mF и высоты 280 км (до которой необходимо учитывать фактор в знаменателе интегральной интенсивности). Полученные таким образом выражения дают возможность точного решения ряда прямых и обратных задач аэрономии, в том числе определения ионосферных параметров N_mF, h_mF и A_0 (IRI-модели) посредством оптических и плазменных измерений.